GCSE Maths – Algebra

Laws of Indices

Notes

WORKSHEET

Laws of Indices

Indices, or an **index**, are another word for powers and are the small floating number that appears after a number or letter. The number which has the index/power applied to it is called the **base**. Indices mean that instead of writing $5 \times 5 \times 5 \times 5$ we can simply write it as 5^4.

There are several rules of indices that need to be remembered:

1. Anything to **the power of 1**, is simply **itself**.

Examples: The power of 1

$$6^1 = 6$$

$$x^1 = x$$

$$(ab)^1 = a^1b^1 = ab$$

2. Anything **to the power of 0** is simply **1**.

Examples: The power of 0

$$x^0 = 1$$

$$3^0 = 1$$

$$68^0 = 1$$

$$e^0 = 1$$

3. **Multiplication**
When **multiplying numbers** with the **same base**, you **add** the powers.

Examples: Multiplying numbers with the same base

$$3^3 \times 3^5 = 3^{3+5} = 3^8$$

$$a^6 \times a^4 = a^{6+4} = a^{10}$$

$$a^3 \times ab = a^{3+1}b = a^4b$$

$$x^2y \times x^{-4}y^2 = x^{2+(-4)}y^{1+2} = x^{-2}y^3$$

4. **Division**
When **dividing numbers** with the **same base**, you **subtract** the powers.

Examples: Dividing numbers with the same base

$$8^7 \div 8^5 = 8^{7-5} = 8^2$$

$$b^9 \div b^2 = b^{9-2} = b^7$$

$$x^3y^{-2} \div x^{-1}y^4 = x^{3-(-1)}y^{-2-4} = x^4y^{-6}$$

Note: For Rule 3 and Rule 4, if we have an expression such as $2^4 \times 3^6$ or $2^4 \div 3^6$ we cannot add or subtract the powers because the **bases are different**. The first number has base 2, whereas the second number has base 3.

5. **Multiplication of powers**
When **raising** one power to another, the powers are **multiplied** together.

Examples: Raising one power to another power

$$(4^2)^4 = 4^{2\times4} = 4^8$$

$$(c^3)^5 = c^{3\times5} = c^{15}$$

6. **Negative powers**
Negative powers are equivalent to reciprocals – the base can be flipped upside down and the power made positive. The general rule is

$$\boldsymbol{a^{-m} = \frac{1}{a^m}}.$$

Examples: Negative powers

$$8^{-2} = \frac{1}{8^2} = \frac{1}{64}$$

$$c^{-3} = \frac{1}{c^3}$$

$$3^{-1} = \frac{1}{3^1} = \frac{1}{3}$$

$$\left(\frac{4}{5}\right)^{-2} = \left(\frac{5}{4}\right)^2 = \frac{5^2}{4^2} = \frac{25}{16}$$

Laws of Indices – Practice Questions

1. Simplify the following:

   a) $8p^4 \times 4p^8$

   b) $7r^6 \times 8s^5 \times 9r^4$

   c) $2^5 \times 2^8 \times a^3$

   d) $9f^4 \times 4^8 \times 2g^8 \times 4^5$

2. Simplify the following:

   a) $x^2y^3 \div xy^2$

   b) $16f^7g^2 \div 4f^3g$

   c) $2r^8s^5t^2 \div r^2s^2$

   d) $21j^8k^3l^3 \div 3k^2l$

3. Simplify the following:

   a) $(8^9)^3$

   b) $(h^4)^{16}$

   c) $(ft^8)^5$

   d) $(c^2d^3)^4$

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.